HOSSAM GHANEM

(8) 4.4 *Limits involving infinity (B)*

Example 1
5 July 13, 1992

Evaluate the following limit $\lim\limits_{x\to-\infty}\dfrac{|x|}{\sqrt{4x^2+3}}$

Solution

$$\lim_{x\to-\infty}\frac{|x|}{\sqrt{4x^2+3}}=\lim_{x\to-\infty}\frac{|x|}{|x|\sqrt{4+\frac{3}{x^2}}}=\lim_{x\to-\infty}\frac{-x}{-x\sqrt{4+\frac{3}{x^2}}}=\lim_{x\to-\infty}\frac{1}{\sqrt{4+\frac{3}{x^2}}}=\frac{1}{\sqrt{4+0}}=\frac{1}{2}$$

Example 2
35 August 15, 2009

Evaluate the following limit $\lim\limits_{x\to-\infty}\dfrac{\sqrt{2x^2+1}}{x+1}$

Solution

$$L=\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{x+1}=\frac{\infty}{-\infty}$$

$$L=\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{x+1}=\lim_{x\to-\infty}\frac{|x|\sqrt{2+\frac{1}{x^2}}}{x+1}=\lim_{x\to-\infty}\frac{-x\sqrt{2+\frac{1}{x^2}}}{x+1}=\lim_{x\to-\infty}\frac{-\sqrt{2+\frac{1}{x^2}}}{1+\frac{1}{x}}=\frac{-\sqrt{2}}{1}=-\sqrt{2}$$

Example 3
6 January 6, 1993

Evaluate the following limit $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{8x^3+1}}{|x|}$

Solution

$$L=\lim_{x\to-\infty}\frac{\sqrt[3]{8x^3+1}}{|x|}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to-\infty}\frac{\sqrt[3]{8x^3+1}}{|x|}=\lim_{x\to-\infty}\frac{x\sqrt[3]{8+\frac{1}{x^3}}}{-x}=\lim_{x\to-\infty}-\sqrt[3]{8+\frac{1}{x}}=-\sqrt[3]{8+0}=-2$$

Example 4
14 January 6, 1996

Evaluate the following limit $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2-1}}{x}$

Solution

$$L=\lim_{x\to\infty}\frac{\sqrt{x^2-1}}{x}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to\infty}\frac{\sqrt{x^2-1}}{x}=\lim_{x\to\infty}\frac{|x|\sqrt{1-\frac{1}{x^2}}}{x}=\lim_{x\to\infty}\frac{x\sqrt{1-\frac{1}{x^2}}}{x}=\lim_{x\to\infty}\sqrt{1-\frac{1}{x^2}}=1$$

Example 5

Evaluate the following limit $\lim_{x\to\infty}\frac{\sqrt{x^2+x}}{2x+1}$

Solution

$$L=\lim_{x\to\infty}\frac{\sqrt{x^2+x}}{2x+1}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to\infty}\frac{\sqrt{x^2+x}}{2x+1}=\lim_{x\to\infty}\frac{|x|\sqrt{1+\frac{1}{x}}}{2x+1}=\lim_{x\to\infty}\frac{x\sqrt{1+\frac{1}{x}}}{2x+1}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x}}}{2+\frac{1}{x}}=\frac{1}{2}$$

Example 6

22 August 11.2001 A

Evaluate the following limit $\lim_{x\to\infty}\left(\frac{\sqrt{4x^2+1}}{x+2}+x\sin\frac{1}{x}\right)$

Solution

$$L=\lim_{x\to\infty}\left(\frac{\sqrt{4x^2+1}}{x+2}+x\sin\frac{1}{x}\right)$$

$$L_1=\lim_{x\to\infty}\frac{\sqrt{4x^2+1}}{x+2}=\frac{\infty}{\infty}$$

$$L_1=\lim_{x\to\infty}\frac{\sqrt{4x^2+1}}{x+2}=\lim_{x\to\infty}\frac{|x|\sqrt{4+\frac{1}{x^2}}}{x+2}=\lim_{x\to\infty}\frac{x\sqrt{4+\frac{1}{x^2}}}{x+2}=\lim_{x\to\infty}\frac{\sqrt{4+\frac{1}{x^2}}}{1+\frac{2}{x}}=\frac{\sqrt{4}}{1}=2$$

$$L_2=\lim_{x\to\infty}x\sin\frac{1}{x}=\infty\cdot 0$$

let $t=\frac{1}{x}$ $\quad\therefore\ as\ x\to\infty,\quad t\to 0$

$$L_2=\lim_{t\to 0}\frac{\sin t}{t}=1$$

$$L=L_1+L_2=2+1=3$$

Example 7

17 January 8 ,1997

Evaluate the following limit $\lim_{x\to\infty}\frac{x^3}{x^2+\sqrt{x^4+x+1}}\sin\frac{1}{x}.$

Solution

$$L=\lim_{x\to\infty}\frac{x^3}{x^2+\sqrt{x^4+x+1}}\cdot\sin\frac{1}{x}=\lim_{x\to\infty}\frac{x^2}{x^2+\sqrt{x^4+x+1}}\cdot x\sin\frac{1}{x}$$

$$L_1=\lim_{x\to\infty}\frac{x^2}{x^2+\sqrt{x^4+x+1}}=\frac{\infty}{\infty}$$

$$L_1=\lim_{x\to\infty}\frac{x^2}{x^2+\sqrt{x^4+x+1}}=\lim_{x\to\infty}\frac{x^2}{x^2+x^2\sqrt{1+\frac{1}{x^3}+\frac{1}{x^4}}}=\lim_{x\to\infty}\frac{1}{1+\sqrt{1+\frac{1}{x^3}+\frac{1}{x^4}}}=\frac{1}{1+1}=\frac{1}{2}$$

$$L_2=\lim_{x\to\infty}x\sin\frac{1}{x}=\infty\cdot 0$$

let $t=\frac{1}{x}$ $\quad\therefore\ as\ x\to\infty,\quad t\to 0$

$$L_2=\lim_{t\to 0}\frac{\sin t}{t}=1\qquad\rightarrow\qquad L=L_1\cdot L_2=\frac{1}{2}(1)=\frac{1}{2}$$

Example 8	Evaluate the following limit $\lim_{x\to 5^-} \frac{3}{2x-10}$

Solution

$\lim_{x\to 5^-} \frac{3}{2x-10} = -\infty$

Example 9	Evaluate the following limit $\lim_{x\to 3^+} \frac{5}{x-3}$

Solution

$\lim_{x\to 3^+} \frac{5}{x-3} = \infty$

Example 10	Evaluate the following limit $\lim_{x\to 4^+} \frac{2}{4-x}$

Solution

$\lim_{x\to 4^+} \frac{2}{4-x} = -\infty$

Example 11	Evaluate the following limit $\lim_{x\to 7^-} \frac{5}{7-x}$

Solution

$\lim_{x\to 7^-} \frac{5}{7-x} = \infty$

Example 12	Evaluate the following limit $\lim_{x\to 2^+} \frac{x+1}{x^2-4}$

Solution

$\lim_{x\to 2^+} \frac{x+1}{x^2-4} = \infty$

Example 13 41 7 January 2012	[2 Pts.] Evaluate the following limit , if it exist $\lim_{x\to 2} \frac{x^2+1}{x-2}$

Solution

$L = \lim_{x\to 2} \frac{x^2+1}{x-2}$

$L_1 = \lim_{x\to 2^-} \frac{x^2+1}{x-2} = -\infty$

$L_2 = \lim_{x\to 2^+} \frac{x^2+1}{x-2} = \infty$

$L_1 \neq L_2$

$\therefore L$ D.N.E

Homework

1	Evaluate the following limit	$\lim\limits_{x\to 3^-} \dfrac{x+1}{x^2-9}$
2	Evaluate the following limit	$\lim\limits_{x\to\infty} x-\sqrt{x^2+1}$

1	Evaluate the following limit $\lim\limits_{x\to 3^-} \dfrac{x+1}{x^2-9}$

Solution

$$\lim_{x\to 3^-} \frac{x+1}{x^2-9} = -\infty$$

2	Evaluate the following limit $\lim\limits_{x\to \infty} x - \sqrt{x^2+1}$

Solution

$$L = \lim_{x\to\infty} \left(x - \sqrt{x^2+1}\right) = \infty - \infty$$

$$L = \lim_{x\to\infty} \frac{\left(x - \sqrt{x^2+1}\right)\left(x + \sqrt{x^2+1}\right)}{\left(x + \sqrt{x^2+1}\right)} = \lim_{x\to\infty} \frac{x^2 - (x^2+1)}{x + |x|\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{-1}{x + |x|\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{-1}{x + x\sqrt{1+\frac{1}{x^2}}} = \frac{-1}{\infty} = -\infty$$